RADIATION LOSSES
IN CURVED DIELECTRIC IMAGE WAVEGUIDES
OF RECTANGULAR CROSS SECTION

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Abstract

An approximate analytical model has been developed for predicting the radiation attenuation in curved rectangular dielectric image waveguide. The radiation is shown to depend on radius of curvature, dielectric constant, and cross-sectional dimensions. Correlation of experimental measurements with theory was good.

Introduction

The need exists for a low loss form of transmission line for integrated circuits operating at millimeter wavelengths. The microstrip configuration, widely used for microwave integrated circuits at X-band and below, becomes increasingly lossy at higher frequencies to the point where circuit functions requiring circuit Q greater than 100-200 cannot be performed. The high permittivity dielectric image waveguide has been proposed to meet this need.[1] Rectangular dielectric image waveguide and the microstrip line are compared in Fig. 1. Guiding in the image waveguide results entirely from refractive effects at the interface between the dielectric and the surrounding medium. Because of this, the narrow center strip, which is responsible for most of the conductive losses in microstrip, is not required. The image plane ohmic losses which do occur in the image plane are lower than in the microstrip ground plane (for comparable frequency and dielectric constant), because the cross-sectional dimensions of the image waveguide are considerably larger than those of microstrip.

Attenuation due to dielectric and conductor losses have been calculated[2], and shown[3] to be more than an order-of-magnitude lower than in comparable microstrip. Since the image line is not enclosed by metal, another possible source of attenuation is radiation due to curvature. The purpose of the present paper is to present the results of an analysis of radiation from curved image waveguides. These results are useful in determining the minimum allowable radius of curvature to be used in waveguides and transmission line devices such as couplers and resonators.

Radius of Curvature Analysis

Figure 2 shows a representation of a curved dielectric image guide (top view). The shaded wave-guiding region has a dielectric constant larger than that of the surrounding region, resulting in a transverse field distribution for the guided mode F(x) which decays exponentially but remains finite. To obtain an approximate expression for radiation loss (dB/radian) as a function of bending radius R, this may be visualized as a two-dimensional guide with an isotropic surrounding region capable of supporting a free-space radiating wave. At some transverse distance x_r, the maintenance of a pure guided mode with equiphase fronts on radial planes requires energy propagating at the speed of light, and for $x > x_r$ a pure guided mode implies energy propagating at greater than the velocity of light. This is true at some value of x_r for any finite bend radius R, since F(x) extends indefinitely in the x-direction. To a first approximation, it is assumed here that the transverse field distribution F(x) is virtually the same in the curved region as in a straight guide for large R. The fraction of energy in the guided mode at $x > x_r$ is assumed to be lost to radiation; this loss is taken to occur in a longitudinal distance equal to the collimated-beam length associated with the field F(x).[4]

It is shown elsewhere[4,5] that the attenuation constant (due to radiation) for the fundamental mode (E_{11}^y) of the bend region is given by

$$\alpha_r = \frac{1}{2Z_c}\frac{E_\ell}{E_T} \qquad (1)$$

where

$$E_\ell = \frac{1}{2k_{xo}} \cos^2 (ak_x) \left[- 2k_{xo}(x_r-a)\right] \qquad (2)$$

$$E_T = a + \frac{\sin(2ak_x)}{2k_x} + \frac{\cos^2(ak_x)}{k_{xo}} \qquad (3)$$

$$Z_c = \frac{2\left[a+\cos(ak_x)/k_{xo}\right]^2}{\lambda_o} \qquad (4)$$

$$x_r = \left(\frac{k_z-k_o}{k_o}\right) R \qquad (5)$$

k_x = propagation constant in the x-direction

$$k_{xo} = \left[(\varepsilon_{re}-1)\ k_o^2\right]^{1/2} \qquad (6)$$

$$\varepsilon_{re}^{*} = \varepsilon_r - \frac{k_y^2}{k_o^2} \quad (7)$$

k_y = propagation constant in the transverse y-direction

$$k_z = \varepsilon_r k_o^2 - k_y^2 - k_x^2 \quad (8)$$

k_o = $2\pi/\lambda_o$

λ_o = free-space wavelength.

The transverse propagation constants k_x and k_y are obtained by solving the following characteristic equations:

$$bk_y = \frac{\pi}{2} - \tan^{-1}\left(\frac{k_y}{\varepsilon_r k_{yo}}\right) \quad (9)$$

$$ak_x = \frac{\pi}{2} - \tan^{-1}\left(\frac{k_x}{k_{xo}}\right) \quad (10)$$

Equations (9) and (10) result from applying the appropriate boundary conditions at the image guide interfaces.

Numerical Results and Experimental Comparison

Equations (1) through (10) were programmed for a digital computer and typical results are shown in Figs. 3 and 4 for an image guide with an aspect ratio $a/b = 1$, $\varepsilon_r = 9$, and $\varepsilon_r = 2.25$, respectively. The cross section of the guide is 2a by b, and the attenuation constant α_r in dB/radian is plotted versus the normalized height of the image guide defined as

$$B = \frac{4b}{\lambda_o}\sqrt{\varepsilon_r - 1} \quad (11)$$

The parameter B becomes the frequency variable for a fixed guide configuration.

Losses were measured from 8-12 GHz in a curved image waveguide. The waveguide is shown in Fig. 5. Measurable curvature attenuation occurred for frequencies below 9.4 GHz. The curvature attenuation, normalized to the total angle of the curves in radians, is shown in Fig. 6. Shown for comparison in Fig. 6 is the curvature attenuation derived from the theoretical analysis. The experimental and theoretical results are in reasonable agreement.

*The use of the equivalent dielectric constant has been introduced by the present authors. Earlier results[1] have shown that this step, which essentially couples the vertical and horizontal field distributions, results in greater accuracy in prediction of guide velocity for low values of B. This is particularly important in the present analysis because this is the range of B (frequency) where curvature radiation occurs.

The displacement to the right of the experimental attenuation curve by some 5.7 percent at the base line (approximately zero radiation) can be attributed to possible experimental deviations from the analytical model. The actual dielectric constant of the waveguide material could deviate from 9.0 by several percent. Also, the physical dimensions of the waveguide could deviate from those assumed by 2-3 percent. Thus, an approximate analysis has been developed to predict the curvature radiation from rectangular dielectric image waveguide for low to moderate values of the radiation constant.

It is also of interest to compare the predicted curvature radiation from image waveguides made from materials having different dielectric constants. An earlier publication on the image waveguide[1] hypothesized that the use of higher dielectric constant would reduce radiation from curved waveguides. A comparison of the results given in Figs. 3 and 4 for the same B shows that the image waveguide having a dielectric constant of 9 will radiate less than one having a dielectroc constant of 2.25. This is shown by the representative data given in the table below for $R/2a = 4$.

	α (dB/radian)	
B \ ε_r	2.25	9.0
1.0	3.3	2.9
1.2	5.8×10^{-1}	2.6×10^{-3}
1.4	7.3×10^{-2}	$< 10^{-6}$
1.6	6.8×10^{-3}	$\ll 10^{-6}$

Therefore, the hypothesis regarding the effect of dielectric constant on guidability of the image waveguide is supported by these analytical results.

References

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4. E.A.J. Marcatili and S. E. Miller, "Improved Relations Describing Directional Control in Electromagnetic Wave Guidance," Bell System Technical Journal, V. 48, pp. 2161-2188, September 1969.

5. E.A.J. Marcatili, "Bends in Optical Dielectric Guides," Bell System Technical Journal, V. 48, pp. 2103-2132, September 1969.

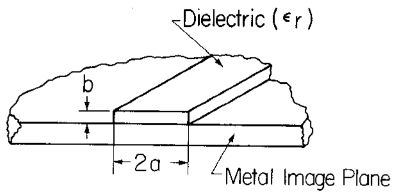

FIG. 1a IMAGE WAVEGUIDE

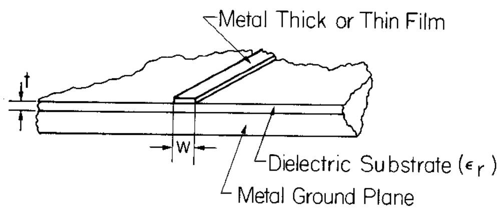

FIG. 1b MICROSTRIP LINE

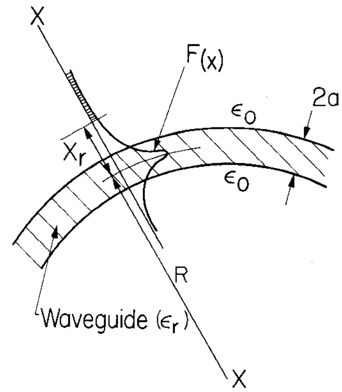

FIG. 2 A CURVED DIELECTRIC WAVEGUIDE

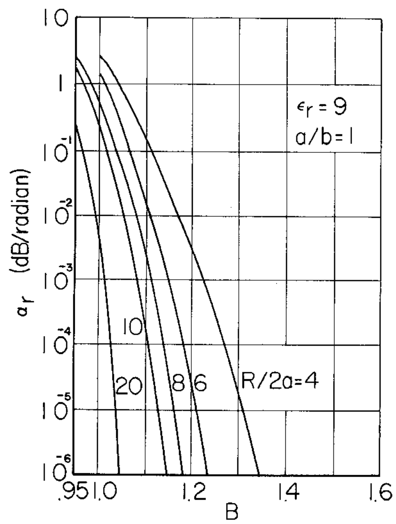

FIG. 3 ATTENUATION CONSTANT DUE TO RADIATION FROM CURVED IMAGE WAVEGUIDE HAVING A DIELECTRIC CONSTANT OF 9.0

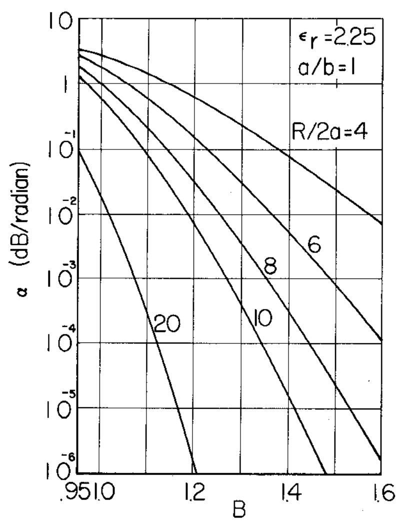

FIG. 4 ATTENUATION CONSTANT DUE TO RADIATION FROM CURVED IMAGE WAVEGUIDE HAVING A DIELECTRIC CONSTANT OF 2.25

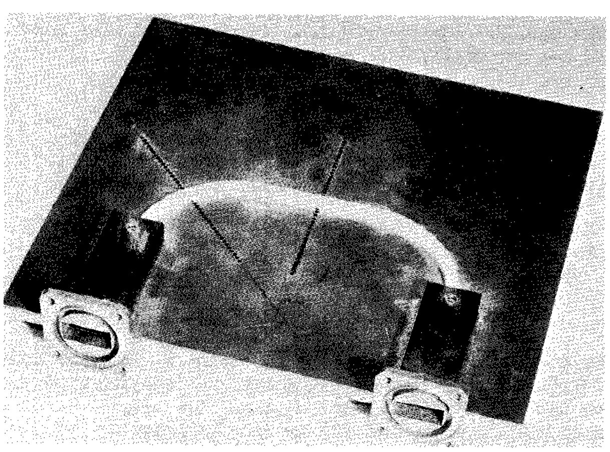

FIG. 5 X-BAND CURVED IMAGE WAVEGUIDE WITH MODE LAUNCHERS

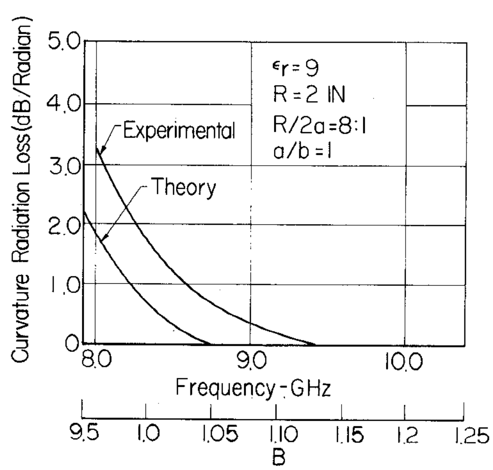

FIG. 6 CURVATURE RADIATION LOSS FOR THE X-BAND WAVEGUIDE